

NUMBER AND PLACE VALUE

<u>Rec</u> Numeral – how to write a number using digits	<u>Year 1</u> Numeral – how to write a number using digits Digit – 24 is a 2-digit number. The 2 represents the tens, the 4 represents the ones Compare - equal (is the same as =), greater, more, less, fewer, Order Sort Count – forwards, backwards, Represent Tens, Ones, One more, One less	<u>Year 2</u> Consecutive – following in order 2,3,4 are consecutive numbers Tens, ones, hundreds Place value Numeral / words Partition Estimate	<u>Year 3</u> Tens, ones, hundreds, thousands Roman numerals 1 – 12 Whole number	<u>Year 4</u> Tens, ones, hundreds, thousands, Tenths, hundredths Whole number Decimal number Decimal point Round to the nearest 10 Round to the nearest 100 Round to the nearest 1,000 Negative numbers – negative 3 is written -3 Roman numerals to 100: I, V, X, L, C	<u>Year 5</u> Tens, ones, hundreds, thousands, ten thousands, hundred thousands, million Tenths, hundredths, Thousandths, Roman numerals to 1,000: I, V, X, L, C, D, M	<u>Year 6</u> Tens, ones, hundreds, thousands, ten thousands, hundred thousands, millions, Tenths, hundredths, Thousandths Decimal places
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Addition

Rec

Part – a number of parts added together makes a whole

Whole – a whole is made up of a number of parts

Equal – symbol (=) read 'equals' or 'is the same as'

Year 1

Numeral – how to write a number using digits

Digit – 24 is a 2-digit number. The 2 represents the tens, the 4 represents the ones

Sum – the total of one or more additions

Total – the sum found by adding

More – to increase an amount

Numberbond – 2 numbers that add together to make a total, e.g. 6+4 is a numberbond to 10.

Adding together – (*aggregation*) – combining 2 parts together

Adding more – (*augmentation*) - starting with an amount and increasing it by another amount

Year 2

Addend – a number to be added to another

Commutative –

addition is commutative so $8 + 2 = 2 + 8$

Inverse – addition and subtraction are inverse operations so $7 + 3 = 10$ and $10 - 3 = 7$

Exchange – when adding the ones in column addition if the total is greater than 10 we exchange 10 ones for a ten OR 10 tens for a hundred.

Bridging 10 – adding 2 numbers to make ten and then add on the rest

Column addition – where the digits are placed in columns to add the numbers together

Year 3

Compensation – a mental strategy where one number is rounded to make the calculation easier and then adjusted e.g. $56 + 38$ is treated as $56 + 40$ and then 2 is subtracted to compensate

Estimate

Increase

Year 4

Consolidation of terms learnt in previous year groups

Year 5

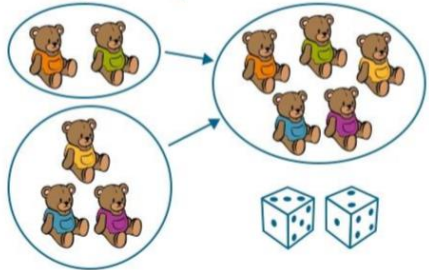
Integer – any of the positive or negative whole numbers

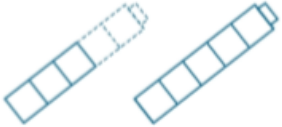
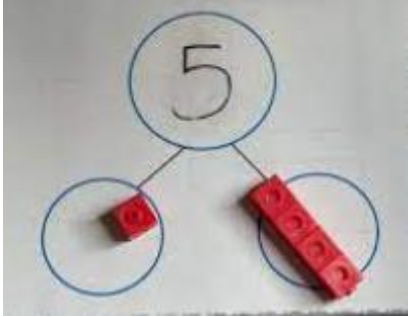
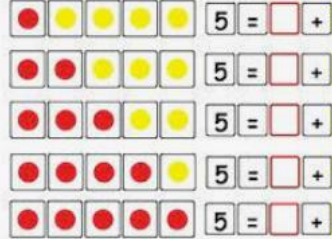
Positive – any number larger than zero

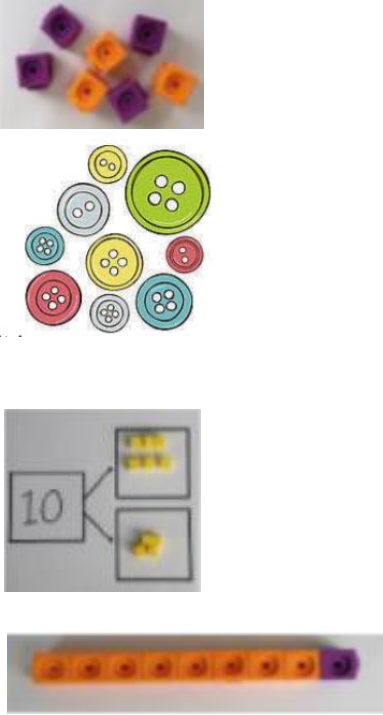
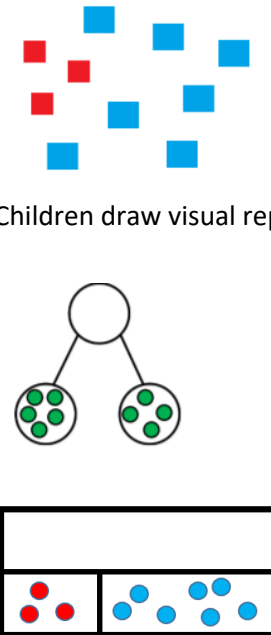
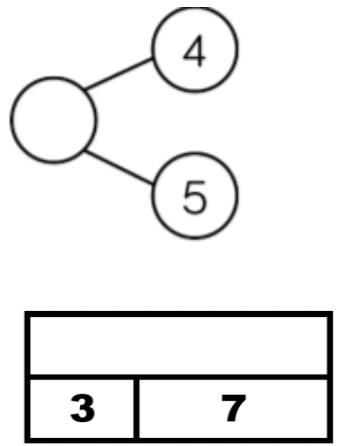
Negative – any number smaller than zero

Year 6

Consolidation of terms learnt in all previous year groups

Objectives and Strategy	Concrete	Pictorial	Abstract	Notes
EYFS				
1 more, one less				
Recognise when one quantity is greater than, less than or the same as another quantity – to 10	Which group of cars would you like to play with? Why? 			
Combining two groups	1. Combining two or more quantities 			

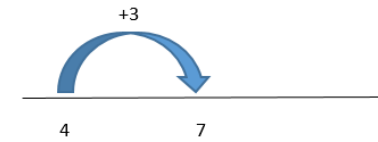
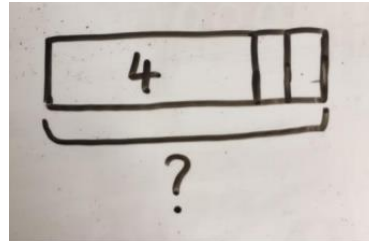
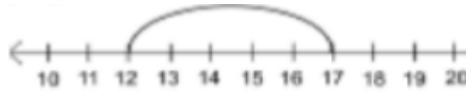
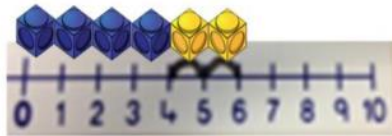
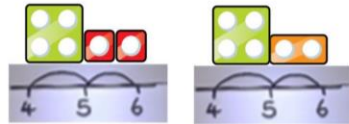
Addition using augmentation	Adding using augmentation (counting on method). This strategy requires fluency with counting from any number. 			
Recall number bonds to 5, then 10	  For number bonds to 10, see Year One.			

Objectives and Strategy	Concrete	Pictorial	Abstract	Notes
Year One				
Two parts to make a whole		 Children draw visual representation.	$7 + 3 =$ 	Using concrete resources to count small collections of objects to a total. Using a part whole model to visually represent two collections coming together into a whole. This is directly linked to the bar model.

Counting
on



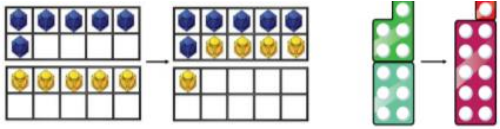
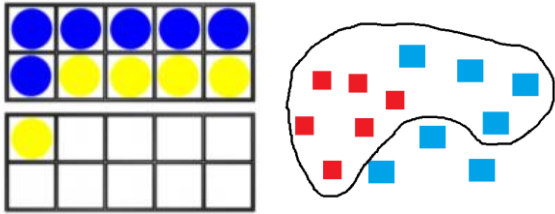
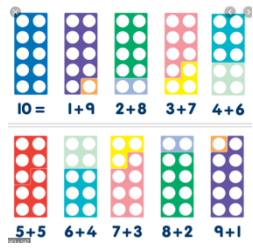
Emphasis on the commutative ability to start with either number, larger being more efficient.

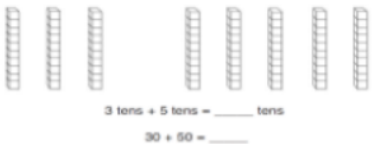
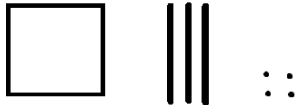
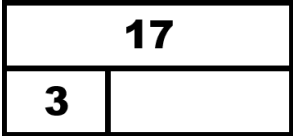
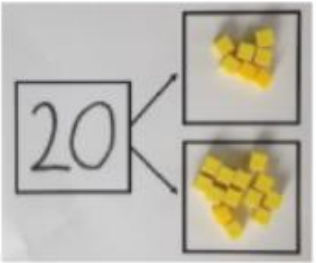
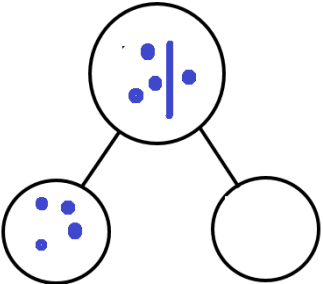


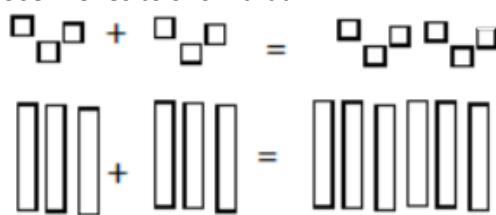
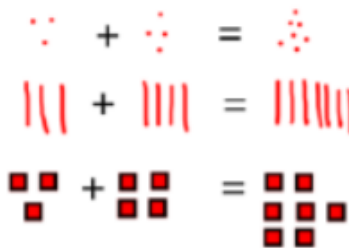
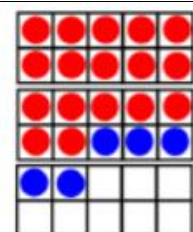
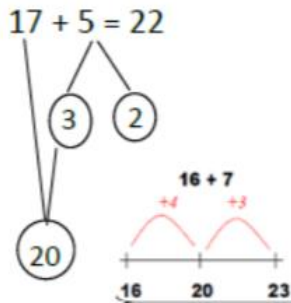
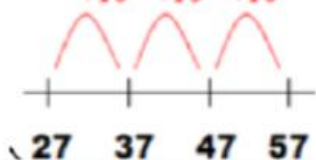
“place the large number in your head and count the smaller number on your hands”

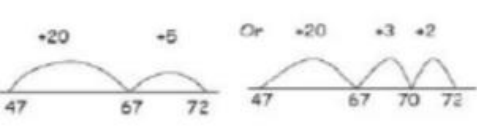
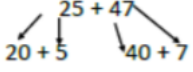
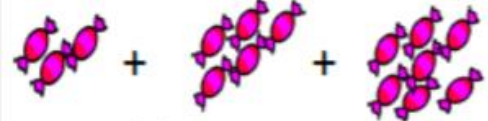
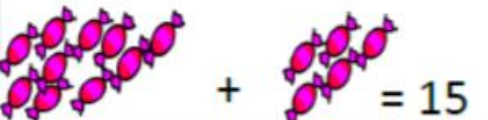
Blank number line as a strategy for jottings.

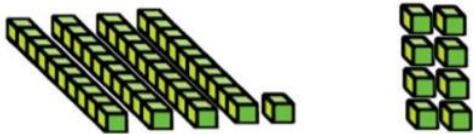
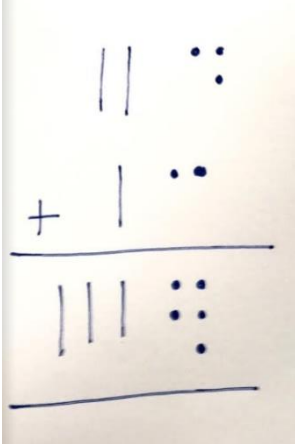
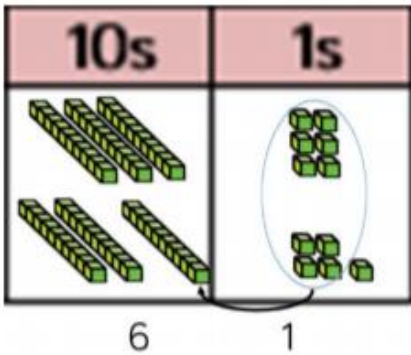
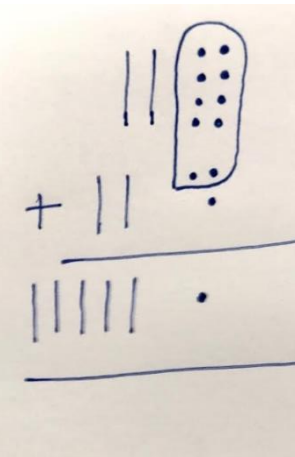
What is 2 more than 4?

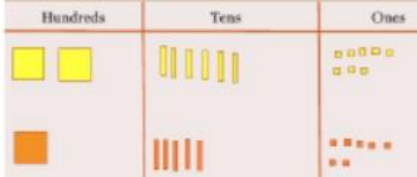
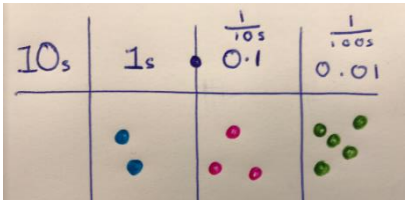
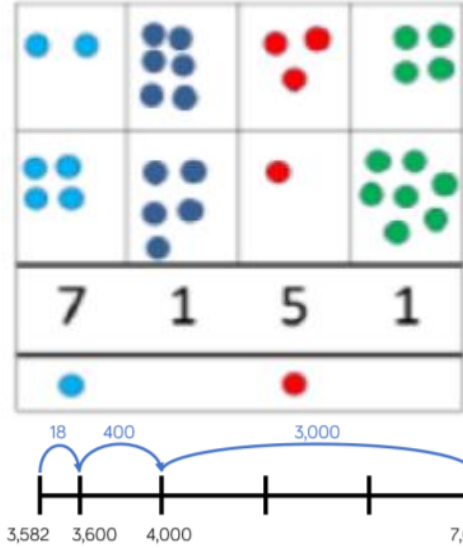
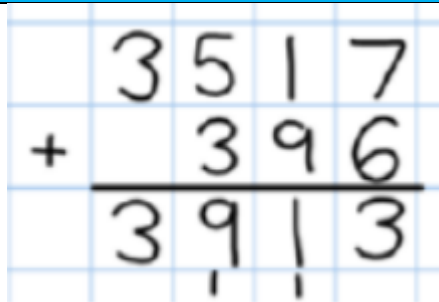
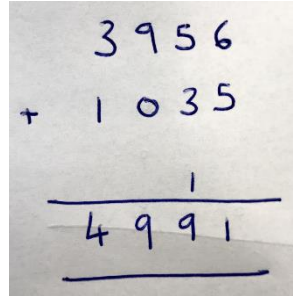
Regrouping to make 10.			$6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$ 	Children to develop an understanding of equality and how the equals sign is a fulcrum of a seesaw.
Number bonds to 10 and 20	 Farer Pete's 10 sheep – youtube. 		Through continued exposure children can recall pairs of number bonds and use these to solve number sentences.	

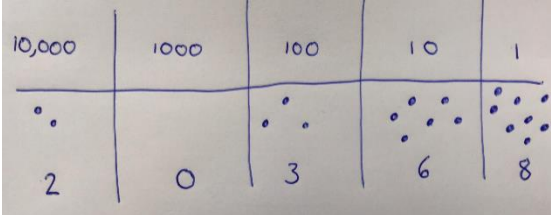
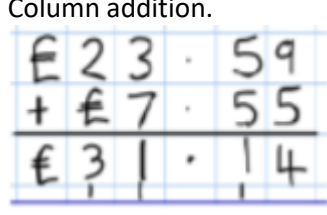
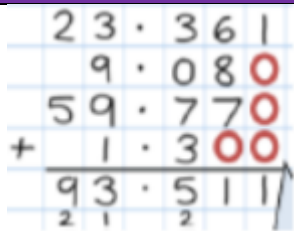
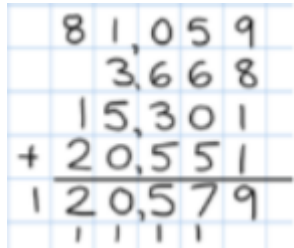
Objectives and Strategy	Concrete	Pictorial	Abstract	Notes
Year Two				
Counting and addition of 10s	 10 20 30 40 50 Counting in 10s using Dienes, beads, counting stick. Counting up, adding one more 10	 3 tens + 5 tens = ____ tens 30 + 50 = ____ Children can draw their own 10s sticks to visually show 	$30 + 40 =$ $50 + \square = 80$	
Solving missing number problems using the inverse. Using the inverse to check calculation.	 If $8 + 1 = 9$ $9 - \square = 8$ Children physically move the resources to demonstrate the inverse.	 Once calculated children display their knowledge of the inverse to create a check e.g. $17 - 3 = 14$ Check: $3 + 14 = 17 \checkmark$	$3 + 14 = 17$ $14 + 3 = 17$ $17 - 3 = 14$ $17 - 14 = 3$	
Solving missing number problems using known number facts	 Investigate the possibilities of making a total		$\square + 4 = 14$	

Using known number facts to solve related problems	Use Dienes to show that: 	Children draw to show that: 	$3 + 4 = 7$ $30 + 40 = 70$					
Bar Model			<table border="1" data-bbox="1431 533 1666 644"><tr><td></td><td></td></tr><tr><td>3</td><td>7</td></tr></table>			3	7	
3	7							
Add two digit numbers and ones	 $17 + 5 = 22$ Use ten frame to make 'magic ten' Children explore the pattern. $17 + 5 = 22$ $27 + 5 = 32$	Use part part whole and number line to model. $17 + 5 = 22$ 	$17 + 5 = 22$ Explore related facts $17 + 5 = 22$ $5 + 17 = 22$ $22 - 17 = 5$ $22 - 5 = 17$ <table border="1" data-bbox="1644 924 1912 1027"><tr><td colspan="2">22</td></tr><tr><td>17</td><td>5</td></tr></table>	22		17	5	
22								
17	5							
Add 2 digit number and tens	 $25 + 10 = 35$ Explore that the ones digit does not change	$27 + 30$ $+10 +10 +10$ 	$27 + 10 = 37$ $27 + 20 = 47$ $27 + \square = 57$					

Add 2 2-digit numbers	 Model using dienes, place value counters and numicon	 Use number line and bridge ten using part whole if necessary.	 $25 + 47$ $20 + 40 = 60$ $5 + 7 = 12$ $60 + 12 = 72$	
Add 3 1-digit numbers	 Combine to make 10 first if possible, or bridge 10 then add third digit	 Regroup and draw representation. 	"Finding secret tens" $4 + 7 + 6 = 10 + 7$ $= 17$ Combine the two numbers that make/ bridge ten then add on the third.	

Objectives and Strategy	Concrete	Pictorial	Abstract	Notes
Year Three				
Column addition with no regrouping	Model using Dienes or numbers. Add ones first, then tens. 		$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ Add the ones first, then the tens, then the hundreds.	
Column addition with regrouping			Looking for ways to make 10. $36 + 25 =$ 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 36 Formal method: $\begin{array}{r} +25 \\ 36 \\ \hline 61 \\ 1 \end{array}$	

Objectives and Strategy	Concrete	Pictorial	Abstract	Notes
Year Four				
Adding numbers up to 4 digits	Children to continue to use pv counters, dienes to add and exchange ones for a ten, tens for a hundred and hundreds for thousands.  	Using counters in the context of a place value table  3,000 + 400 + 18 = 3,418	 An alternative method for those who forget the carried digits when adding 	We advocate using a variety of vocabulary to ensure familiarity with exchange. e.g – exchange, carry, swap ect

Objectives and Strategy	Concrete	Pictorial	Abstract	Notes
Year Five				
Adding numbers over 4 digits	Using concrete representations to show numbers greater than 4 digits allows pupils to consolidate understanding of PV before applying to column addition 		Column addition. 	
Year Six				
Adding several numbers of increasing complexity	As per year 5		 	Draw on all previous experiences. No excuses.

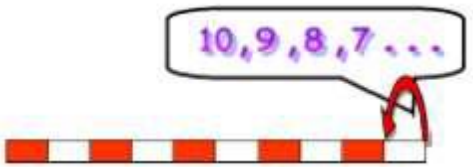
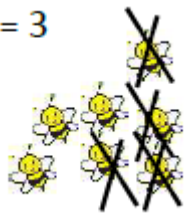
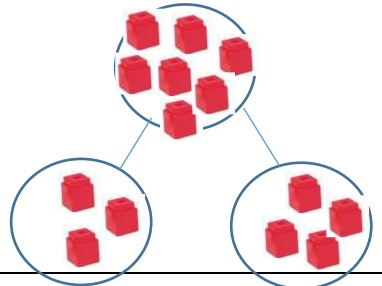
Subtraction

<u>Rec</u> Whole – a whole subtract any number of parts equals a part Take away – to remove a number of items from a group	<u>Year 1</u> Subtract – to carry out the process of subtraction Minus – a name for the symbol ‘-’ Less – to decrease an amount Counting back Finding the difference	<u>Year 2</u> Inverse – addition and subtraction are inverse operations so $10 - 4 = 6$ and $6 + 4 = 10$ (it is NOT commutative) Exchange – when the number to subtract is larger than the number we are subtracting from we exchange a ten into ten ones. Difference – we subtract to find the difference	<u>Year 3</u> Subtrahend – a number to be subtracted from another Minuend – a number from which another is to be subtracted Minuend – Subtrahend = Difference Compensation – a mental strategy where one number is rounded to make the calculation easier and then adjusted e.g. $56 - 38$ is treated as $56 - 40$ and then 2 is added to compensate Efficient subtraction (Y4) – instead of $4,000 - 2,124$ do $3,999 - 2,123$ Decrease	<u>Year 4, 5 & 6</u> Consolidation of terms learnt in previous year groups
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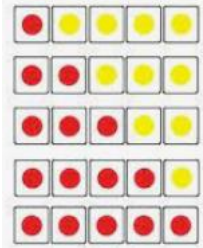
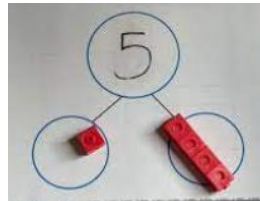
Objectives and Strategy:	Concrete	Pictorial	Abstract	Notes
Nursery				
To make comparisons between quantities	Which group of cars would you like to play with? Why? 			
To use the language of fewer (less) to compare a set of objects.	To use the language of fewer (less) to compare a set of objects.  I have fewer sweets than Jenny.			
To separate a group of 3 or 4 objects in different ways (total still the same)	How many different ways can we put four teddies in two beds? 			
To know that a group of objects changes amounts when something is taken away.	To know that a group of objects changes amount when something is taken away. 			

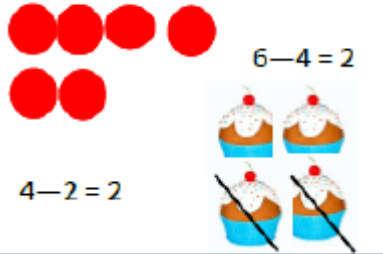
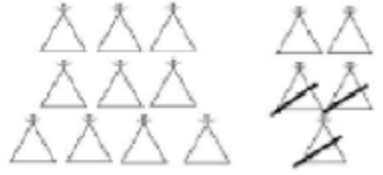
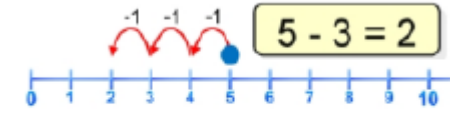
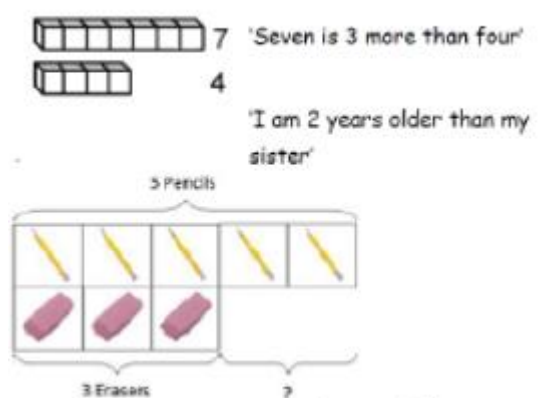
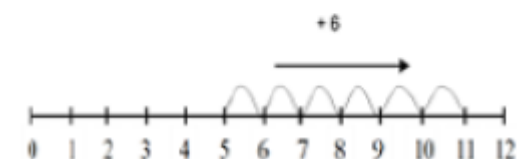
To find the total number of items after some are taken away by counting all of them.	To find the total number of items after some are taken away by counting all of them. 		
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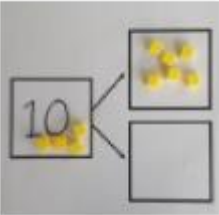
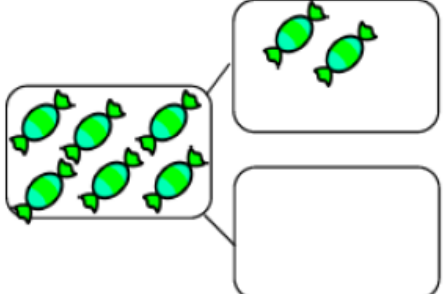
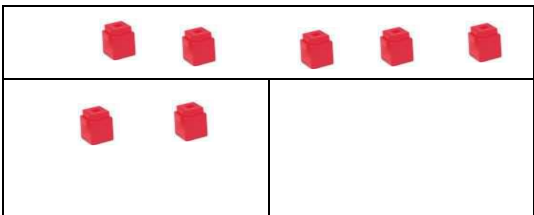
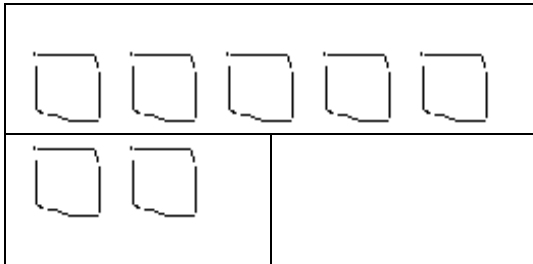
Objectives and Strategy	Concrete	Pictorial	Abstract	Notes
Reception				
Recognise when one quantity is greater than, less than or the same as another quantity – to 10	Which group of cars would you like to play with? Why? 			
Relates subtraction to taking away.	3 toy cars Take away 2 Leaves 1 toy car. 			
To find one less than a group of up to five, then ten objects.	 1 less than 6 is 5	Recording using marks they can explain and apply meaning.  Could be explained using mark-making rather than symbols or numerals.		
Using quantities and objects to subtract single-digit numbers and count on to find the answer.	 $5 - 3 = 2$  $4 - 3 = 1$			

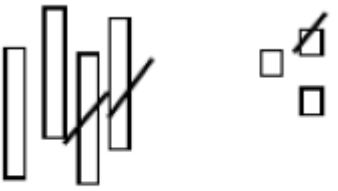
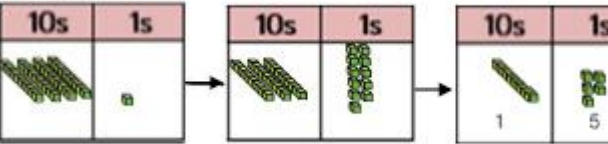
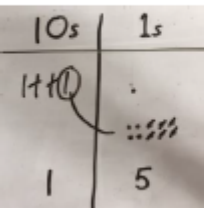
	 Able to show with different concrete resources.			
To count backwards on a number line or counting stick.				
To recognise and name - and = symbols.	To arrange a subtraction calculation with concrete resources.  $7 - 3 = 4$	To solve a subtraction calculation using mark-making. $7 - 4 = 3$ 	To read a subtraction calculation. To read aloud $7 - 4 = 3$ knowing the correct vocabulary for the symbols - and =	
Progression towards bar model	Use objects placed in a part whole model to show a subtraction calculation. 	Use mark-making within a part whole template, using concrete model for support.		

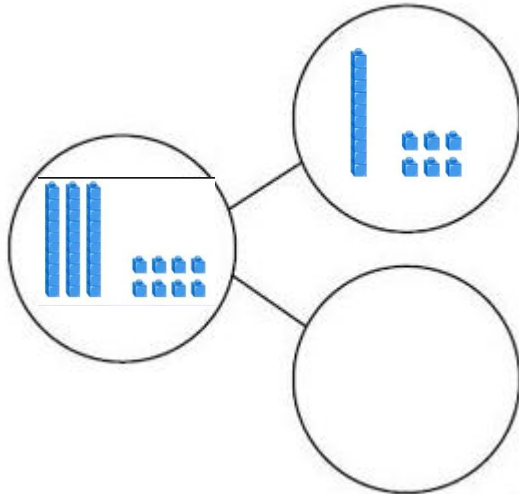
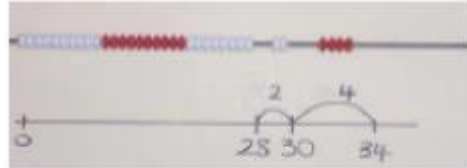
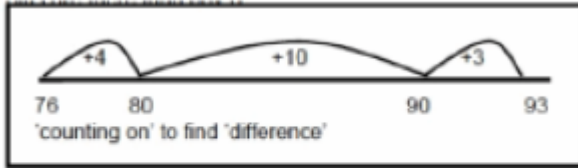
To recall
number bonds
to 5, including
subtraction
facts

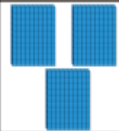
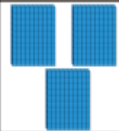
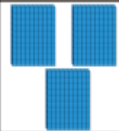
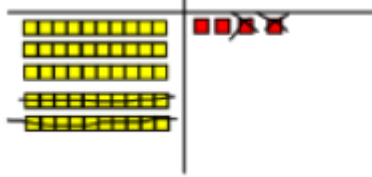
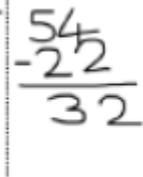
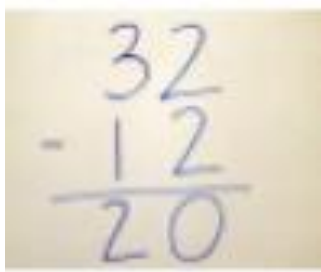


Objectives and Strategy	Concrete	Pictorial	Abstract	Notes
Year One				
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  $6 - 4 = 2$ $4 - 2 = 2$	 $15 - 3 = 12$ Cross out drawn objects to show what has been taken away.	$7 - 4 = 3$ $16 - 9 = 7$	
Counting back	 Move objects away from the group, counting backwards.	 Count back in ones using a number line.	Think of the number 9. You count backwards 4. What number will you be at? Put 13 in your head, then count back 5. Where are you at now?	
Find the difference	Compare objects and amounts  Lay objects to represent bar model.	Count on using a number line to find the difference. 	Hannah has 12 sweets and her sister has 7. How many more does Hannah have compared to her sister?	

Represent and use number bonds and related subtraction facts within 20 Part Whole Model	 Link to addition. Use Part-Whole model to show the inverse. 'If 10 is the whole and 6 is one of the parts, what will the other part be?' $10 - 6 = 4$	 Use pictorial representations to show the part.	Move to using numbers within the part whole model. 	
Bar Model	 Using concrete resources, place them on a bar model template, and describe how many more need to be added to match the top bar.	Use simplistic representations of concrete objects, drawing them in the correct place on a bar model template. 	 $10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$	

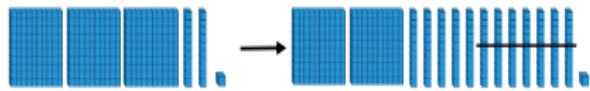
Objectives and Strategy	Concrete	Pictorial	Abstract	Notes
Year Two				
Partitioning to subtract without regrouping	 Use Base 10 to show how to partition the number when subtracting without regrouping.	 $43 - 21 = 22$ Children to draw representations of base 10 to support understanding.	$43 - 21 = 22$	
Partitioning to subtract with regrouping	Column method using base 10 and having to exchange. 41 - 26 	Represent the base 10 pictorially, remembering to show the exchange. 	The logical abstract progression would be column subtraction, but this will come in for the Year 3 subtraction. For now, we would expect Year 2 to draw representations of Base 10, or use a number line, and this is enough.	

Conceptual Variation	 Using a Part-Whole model with Base 10.	Progressing to representing subtraction problems with drawn bar models. <table><tr><td>38</td><td></td></tr><tr><td>16</td><td></td></tr></table>	38		16		Missing number problems using numbers less than 100. Tests understanding of inverse. 38 – ____ = 16	
38								
16								
Using tens as a bridging point	 34 – 28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	93 – 76 = 17					

Objectives and Strategy	Concrete	Pictorial	Abstract	Notes										
Year Three														
Mental subtraction, 3 digit and ones, tens, hundreds	<table><tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr><tr><td></td><td></td><td></td></tr></table> Use place value counters to complete the number sentences. 352 + 4 tens = ____ 352 – 2 tens = ____ Use concrete resources to support working for mental questions.	Hundreds	Tens	Ones				Dora uses the part-whole model and number line to solve 132 – 4  Use this method to calculate: 132 – 8 123 – 8 123 – 5	 Complete: <table><tr><td>793 – 60 =</td></tr><tr><td>793 – 70 =</td></tr><tr><td>793 – 80 =</td></tr><tr><td>793 – 90 =</td></tr></table> If pupils know the mental answer to the first question, they can use the answer to formulate the following answers, recognising the pattern and answering it mentally.	793 – 60 =	793 – 70 =	793 – 80 =	793 – 90 =	
Hundreds	Tens	Ones												
														
793 – 60 =														
793 – 70 =														
793 – 80 =														
793 – 90 =														
Formal Column with 3 digits, no regrouping	 47 – 32  — Use Base 10, Numicon or place value counters to model and show this.	 Draw representations to support understanding 												

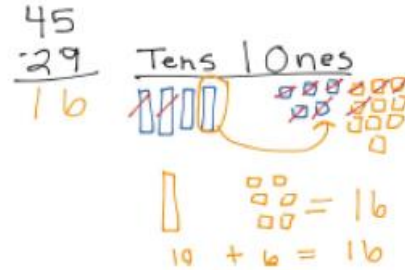
Formal
Column with
3 digits, with
regrouping

Rosie uses Base 10 to subtract 70 from 321

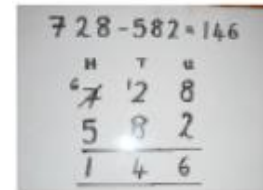


Use Base 10 to represent the answer.

This kind of question could also be calculated and proven using place value counters.

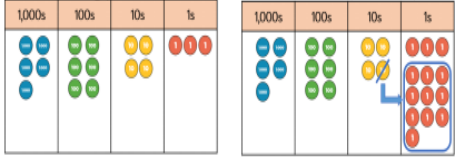
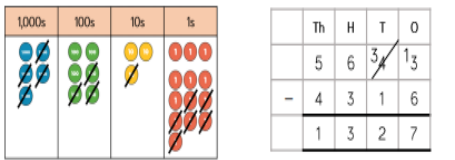
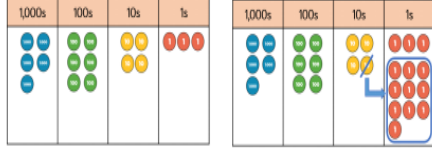


Children may draw base ten or PV counters and cross off.



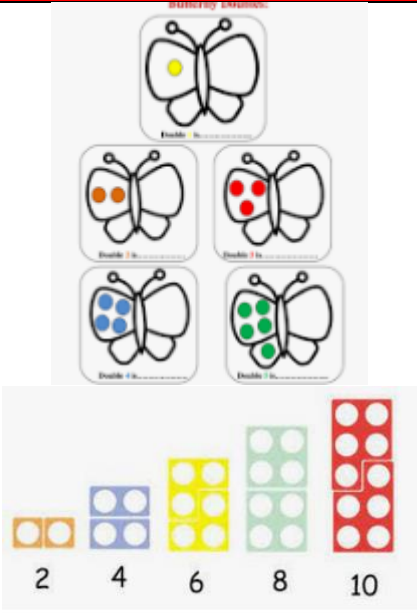
Then move to
formal method.

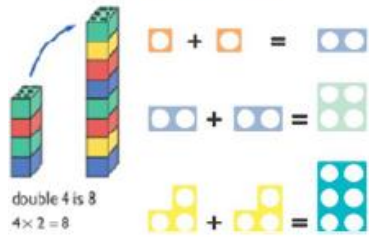
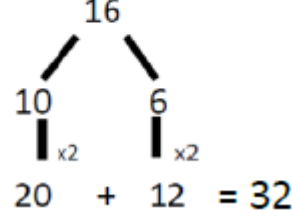
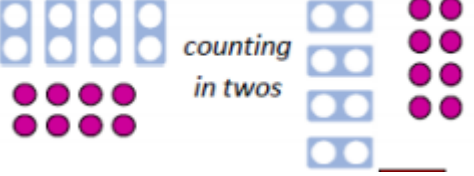
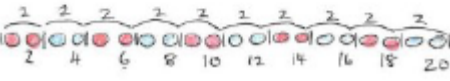
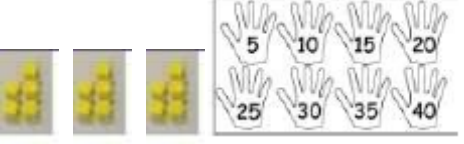
DROITWICH MATHS CLUSTER CALCULATION POLICY 2024

Objectives:	Concrete	Pictorial	Abstract	Notes
Year Four				
4 digit columnar	Dexter is using place value counters to calculate $5,643 - 4,316$   Students can use Base 10 and Place Value counters to represent and describe column subtraction where exchanging is necessary.	Dexter is using place value counters to calculate $5,643 - 4,316$   Rather than physically moving counters, children draw and cross out place value counters to represent the exchange. Same for Base 10.	 Abstract column subtraction, laid out as above.	
Objectives and Strategy	Concrete	Pictorial	Abstract	Notes
Year Five				
Year Six				
4+ digit columnar	Concrete resources would remain the same as Year 4, see above. Year 5 and 6 questions would involve numbers up to 10 million.	Pictorial representations would remain the same as Year 4, see above. Year 5 and 6 questions would involve numbers up to 10 million.	Abstract method would remain the same as Year 4, see above. Year 5 and 6 questions would involve numbers up to 10 million.	

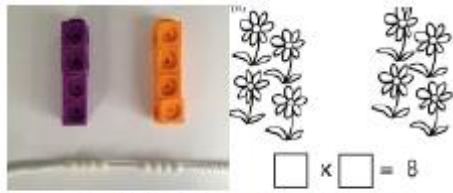
Multiplication

Rec	<u>Year 1</u> Groups of, sets of, lots of Equal groups Counting patterns (2s, 5s, 10s) Doubles	<u>Year 2</u> Multiply – to carry out the process of multiplication Multiple – a number in a times table e.g. the multiples of 2 are 2,4,6 etc. Groups of, lots of, sets of, times, multiplied by – different ways to say the symbol “x” Array – an ordered collection of objects in rows and columns Commutative – knowing 3 x 5 will get the same answer as 5 x 3 Even – numbers in the 2 times table Odd – numbers not in the 2 times tables Pairs	<u>Year 3</u> Factor – factor x factor = product Product – the result of multiplying 2 numbers Multiply Scaling – to enlarge a number, quantity or measurement by an amount	<u>Year 4</u> Factor – factor x factor = product e.g. 1,2,3,4,6,12 are factors of 12 Factor pairs - A <u>factor pair</u> is 2 factors multiplied together to make a given product Short multiplication – a method used to multiply 2 or more digits by a 1 digit number, using columns	<u>Year 5 & 6</u> Prime number – A whole number greater than 1 that only has two factors, itself and 1. Composite – a non prime number. Common factor – a number which is a factor of 2 or more other numbers e.g. 3 is a common factor of 9 and 30, 7 is a common factor of 14 and 21. Prime factor – the factors of a number that are prime e.g. 2 and 3 are the prime factors of 12 Common multiple – the smallest positive number that is a multiple of two or more numbers e.g. 24 is a common multiple of 4,6,8 etc. Square numbers Cube numbers
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Objectives and Strategy	Concrete	Pictorial	Abstract	Notes
EYFS				
Doubling – to 5	 The concrete resources include five butterfly cards and five dot markers. The butterfly cards are arranged in a pyramid: one at the top, two in the middle, and two at the bottom. Each card has a label 'Double' followed by a number and a blank line. The dot markers are arranged in a row below the butterfly cards, each with a label below it: 2 (orange), 4 (blue), 6 (yellow), 8 (green), and 10 (red).			

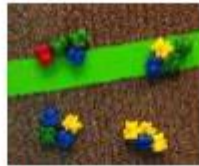
Objectives and Strategy				
Year One				
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling  double 4 is 8 $4 \times 2 = 8$	Draw pictures to show how to double numbers Double 4 is 8 	Partition a number and then double each part before recombining it back together. 	
Counting in Multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting.  counting in twos 	 Children make representations to show counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30	
Making Equal groups and				

counting the
total

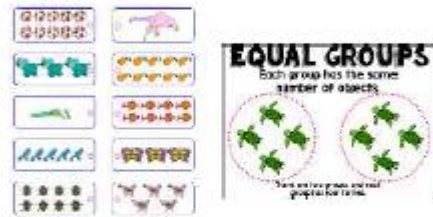


Use manipulatives to create equal groups.

Children can lay out equal groups.



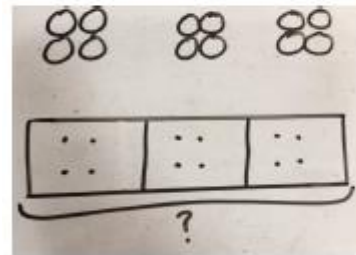
Can match equal groups.



Recognise when they are given equal amounts.



Children to represent the practical resources in a picture and use a bar model.

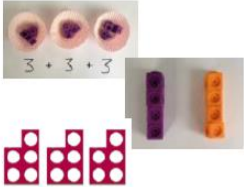
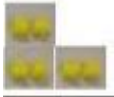
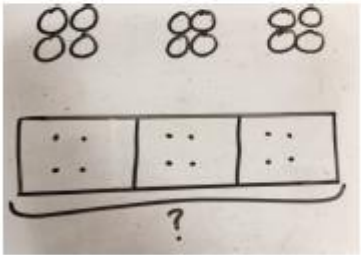
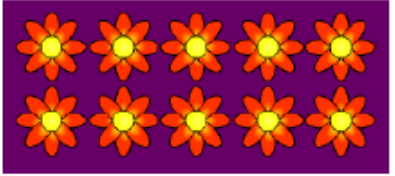
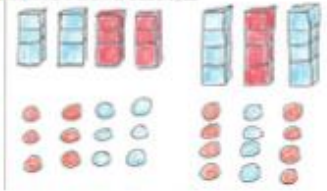


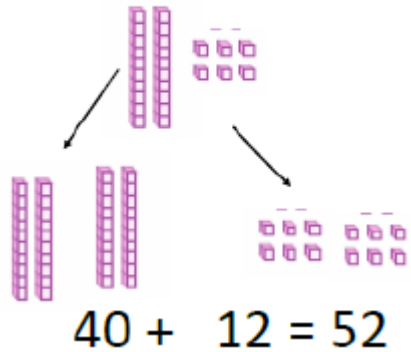
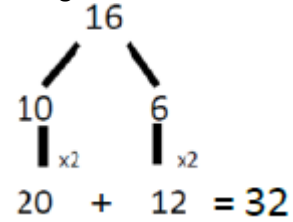
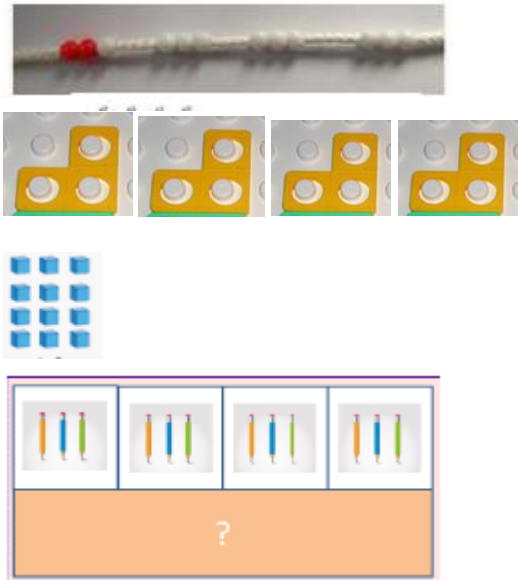
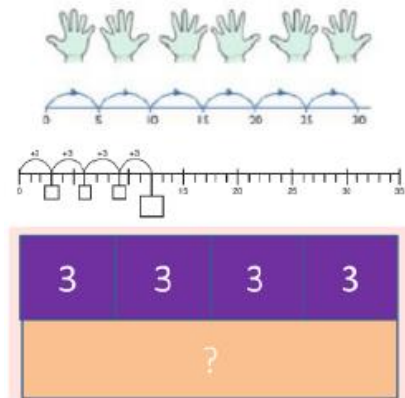
Draw  to show $2 \times 3 = 6$

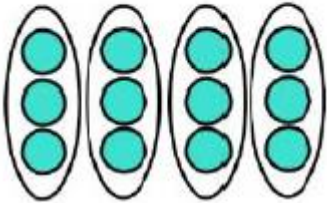
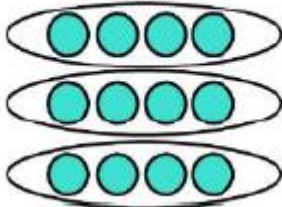
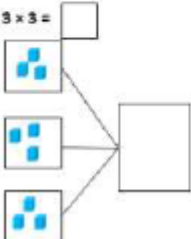
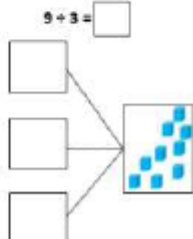
Draw and make representations

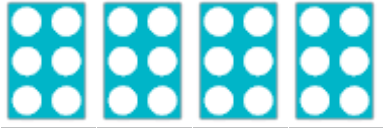
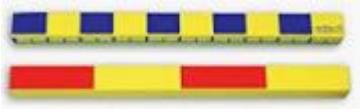
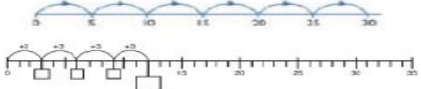
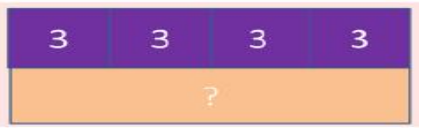
$$3 \times 4 = 12$$

$$4 + 4 + 4 = 12$$

Repeated Addition	 Use different objects to add equal groups  Use Base 10	Children to represent the practical resources in a picture and use a bar model. 	 $2+2+2+2 = 8$ $4 \times 2 = 8$	
Understanding arrays	Use arrays to illustrate commutativity counters and other objects can also be used. $2 \times 5 = 5 \times 2$  2 lots of 5 5 lots of 2 Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding. 	Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$	

Objectives & Strategy	Concrete	Pictorial	Abstract	Notes
Year Two				
Doubling	Model doubling using dienes and PV counters.  $40 + 12 = 52$	Draw pictures and representations to show how to double numbers.	Partition a number and then double each part back before recombining it back together.  $20 + 12 = 32$	
Counting in multiples of 2,3,4,5,10 from 0		Number lines, counting sticks and bar models should be used to show representation of counting in multiples. 	Count aloud in multiples of a number Write sequences with multiples of numbers 0,2,4,6,8, 0,3,6,9,12 0,4,8,12,16 $4 \times 3 =$	
Multiplication is commutative				

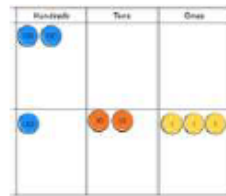
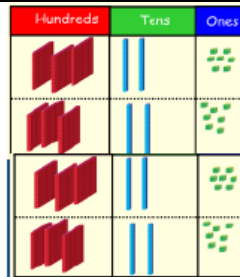
	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	Use an array to write multiplication sentences and reinforce repeated addition.  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$	
Using the inverse	Use of part-part-whole model to establish the inverse relationship between multiplication and division This link should be made explicit from early on, using the language of the part-part-whole model, so that pupils develop an early understanding of the relationship between multiplication and division. Bar models (with Cuisenaire rods) should be used to identify the whole, the value of the parts and the number of parts. It is important to highlight that with multiplication, the parts are of equal value as this is different to how this model is used for addition and subtraction. There are three equal parts. Each part has a value of three. What is the whole?  	 $\square \times \square = \square$ $\square \times \square = \square$ $\square \div \square = \square$ $\square \div \square = \square$	$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 4 = 2$ $8 \div 2 = 4$	

Objectives & Strategy	Concrete	Pictorial	Abstract	Notes
Year Three				
Doubling to derive new multiplication facts	 $2 \times 3 = 6$  $1 \times 4 = 4$ $2 \times 4 = 8$  $= 4 \times 4 = 16$	This builds on the doubling strategy from Year 2. $3 \times 3 = 9$ $3 \times 6 = \text{double } 9 = 18$	$3 \times 4 = 12$ $6 \times 4 = 24$ Counting in steps	
Counting in multiples of 2,3,4,5,6,8,10	  	 Number lines, counting sticks and bar models should be used to show representation of counting in multiples.   	$3, 6, 9, 12, 15, 18$ $1 \times 3 = 3$ $2 \times 3 = 6$ $3 \times 3 = 12$ $4 \times 3 = 15$	

Objectives & Strategy	Concrete	Pictorial	Abstract	Notes
10 x 100x times greater	$5 \times 1 = 5$ $5 \times 10 = 50$ $3 \times 1 = 3$ $3 \times 100 = 300$ $5 = 1 \times 5$ $50 = 10 \times 5$ $500 = 100 \times 5$		10 x bigger than 23 is 10 times greater than 6 is 6 x 60 = 360	The language of 10 times greater must be well modelled and understood to prevent the numerical misconception of adding a zero.
Partitioning Grid Method				

Objectives & Strategy	Concrete	Pictorial	Abstract	Notes						
Year Four										
Multiplying by 10 and 100	Thousands 3 Ten-thousands 3 Hundreds 3 Tens 3 Ones 3 $3 \times 10 = 30$ $3 \times 100 = 300$ $3 \times 1000 = 3000$ factor $3 \times 7 = 21$ product factor $7 \times 3 = 21$ product $30 \times 7 = 210$ $70 \times 3 = 210$ $7 \times 30 = 210$ $3 \times 70 = 210$ $300 \times 7 = 2100$ $700 \times 3 = 2100$ $7 \times 300 = 2100$ $3 \times 700 = 2100$ <td>When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc. When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater. Repeated multiplication by ten will build an understanding of multiplying by 100 and 1000</td>	When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc. When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater. Repeated multiplication by ten will build an understanding of multiplying by 100 and 1000								
Partitioning and Distributive law.	Fill each row with 126 <table><tr><td>x</td><td>100</td><td>20</td><td>6</td></tr><tr><td>4</td><td></td><td></td><td></td></tr></table> <td>As well as partitioning in tens and ones pupils begin to explore compensating strategies and factorisation to find the most efficient solution to a calculation.</td>	x	100	20	6	4				As well as partitioning in tens and ones pupils begin to explore compensating strategies and factorisation to find the most efficient solution to a calculation.
x	100	20	6							
4										

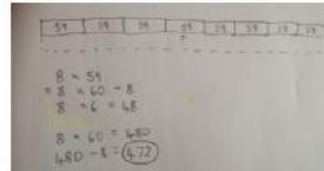
Column Multiplication



It is important at this stage that they always multiply the ones first.

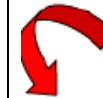
x	300	20	7
4	1200	80	28

The grid method may be used to show how this relates to a formal written method.



Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.

$$\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}$$



$$\begin{array}{r} 327 \\ \times 4 \\ \hline 1308 \end{array}$$

This may lead to a compact method.

$$\begin{array}{r} 241 \\ \times 3 \\ \hline 723 \\ 1 \end{array}$$

Multiplying 3 digit numbers by 1. Associative Law

Four pots each containing two flowers which each have seven petals. How many petals in total?

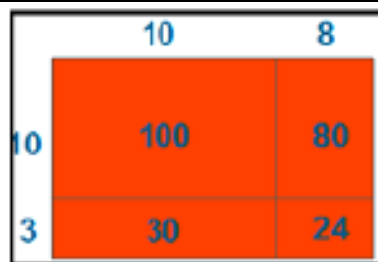


$$(4 \times 2) \times 7 \text{ or } 4 \times (2 \times 7)$$

Objectives & Strategy	Concrete	Pictorial	Abstract	Notes																																																	
Year Five																																																					
Year Six																																																					
Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.	<table><thead><tr><th>Thousands</th><th>Hundreds</th><th>Tens</th><th>Ones</th><th>Tenths</th><th>Hundredths</th></tr></thead><tbody><tr><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td></tr></tbody></table> To visualise and notice what happens to the digits. <table><tr><td>x</td><td>100.</td><td>10.</td><td>1.</td><td>0.1</td><td>0.01</td></tr><tr><td></td><td></td><td></td><td>1.</td><td>6</td><td>6</td></tr><tr><td></td><td></td><td>1</td><td>6.</td><td>6</td><td></td></tr><tr><td></td><td>1</td><td>6</td><td>6</td><td></td><td></td></tr></table>	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths													x	100.	10.	1.	0.1	0.01				1.	6	6			1	6.	6			1	6	6			<table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table> 327 x 4 ----- 28 80 1200 ----- 1308 327 x 4 ----- 1308 1 2 This will lead to a compact method.	x	300	20	7	4	1200	80	28	Avoid saying you add a zero when multiplying by 10 and instead use the language of place holder.
Thousands	Hundreds	Tens	Ones	Tenths	Hundredths																																																
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Column Multiplication for 3 and 4 x 1 digit.	<table><thead><tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr></thead><tbody><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></tbody></table> It is important at this stage that they always multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642 327 x 4 ----- 28 80 1200 ----- 1308 327 x 4 ----- 1308 1 2 This will lead to a compact method.	Hundreds	Tens	Ones																																																	
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Deriving from a known fact				Explore links between multiplication facts and x 10, 100, 1000																																																	

	These are the multiplication facts pupils should be able to derive from a known fact <table border="1"> <tr> <td>2 100 000</td><td>700 000 × 3</td><td>70 000 × 30</td><td>7000 × 300</td><td>700 × 3000</td><td>70 × 30 000</td><td>7 × 300 000</td></tr> <tr> <td>210 000</td><td>70 000 × 3</td><td>7000 × 30</td><td>700 × 300</td><td>70 × 3000</td><td>7 × 30 000</td><td></td></tr> <tr> <td>21 000</td><td>7000 × 3</td><td>700 × 30</td><td>70 × 300</td><td>7 × 3000</td><td></td><td></td></tr> <tr> <td>2100</td><td>700 × 3</td><td>70 × 30</td><td>7 × 300</td><td></td><td></td><td></td></tr> <tr> <td>210</td><td>70 × 3</td><td>7 × 30</td><td></td><td></td><td></td><td></td></tr> <tr> <td>21 = 7 × 3</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr> <td>2.1</td><td>0.7 × 3</td><td>7 × 0.3</td><td></td><td></td><td></td><td></td></tr> <tr> <td>0.21</td><td>0.07 × 3</td><td>0.7 × 0.3</td><td>7 × 0.03</td><td></td><td></td><td></td></tr> <tr> <td>0.021</td><td>0.007 × 3</td><td>0.07 × 0.3</td><td>0.7 × 0.03</td><td>7 × 0.003</td><td></td><td></td></tr> </table>					2 100 000	700 000 × 3	70 000 × 30	7000 × 300	700 × 3000	70 × 30 000	7 × 300 000	210 000	70 000 × 3	7000 × 30	700 × 300	70 × 3000	7 × 30 000		21 000	7000 × 3	700 × 30	70 × 300	7 × 3000			2100	700 × 3	70 × 30	7 × 300				210	70 × 3	7 × 30					21 = 7 × 3							2.1	0.7 × 3	7 × 0.3					0.21	0.07 × 3	0.7 × 0.3	7 × 0.03				0.021	0.007 × 3	0.07 × 0.3	0.7 × 0.03	7 × 0.003			Use doubling and halving of numbers here..
2 100 000	700 000 × 3	70 000 × 30	7000 × 300	700 × 3000	70 × 30 000	7 × 300 000																																																															
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Knowledge of factors	6 × 2 × 12 6 × 12 × 2 Three and eight are factors of 24: 6 × 3 × 8 6 × 3 × 8 6 × 8 × 3 Four and six are factors of 24: 6 × 4 × 6 6 × 4 × 6 6 × 6 × 4					Calculate 6 × 23 using factor pairs Two and twelve are factors of 24 6 × 2 × 12 or 6 × 12 × 2 Three and eight are factors of 24 6 × 3 × 8 or 6 × 8 × 3 Four and six are factors of 24 6 × 4 × 6 or 6 × 6 × 4																																																															

Long multiplication



	1	8
×	1	3
	5	4
	1	8
	2	3

18 x 3 on the first row
(8 x 3 = 24, carrying the 2 for 20, then 1 x 3)
18 x 10 on the 2nd row. Show

multiplying by 10 by putting zero in units first

$$\begin{array}{r} 1234 \\ \times 16 \\ \hline 7404 \\ 12340 \\ \hline 19744 \end{array}$$

(1234 x 6)
(1234 x 10)

Multiplying decimals up to 2 decimal places by a single digit.

Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.

$$\begin{array}{r} 3.19 \\ \times 8 \\ \hline 25.52 \end{array}$$

Division

Rec & Year 1

Equal – the same

Sharing – share equally a number of objects into a specified number of groups.

Divide – to carry out the process of division

Make equal groups – grouping

Make equal groups - sharing

Year 2

Sharing – sharing equally between

Grouping - put into groups of

Divided by – sharing or grouping

Inverse – multiplication and division are inverse operations so $10 \div 2 = 5$ and $5 \times 2 = 10$ (it is NOT commutative)

Even – numbers that can be divided by 2

Odd – numbers that will have a remainder of 1 when divided by 2

Year 3, 4, 5 and 6

Dividend – the number that is being divided into equal parts

Divisor – for sharing: the number that it is being shared between. For grouping: the number in each group In $15 \div 3$, 15 is the dividend and 3 is the divisor

Quotient – the result of a division

$\text{dividend} \div \text{divisor} = \text{quotient}$

Divisible – A whole number is divisible by another if there is no remainder after division

Remainder – the amount remaining after division

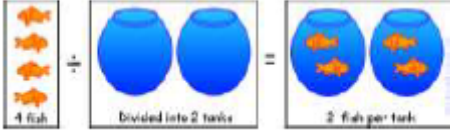
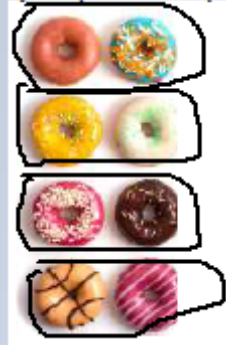
e.g. $29 \div 7 = 4 \text{ r}1$

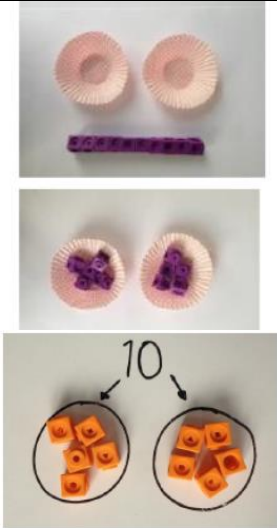
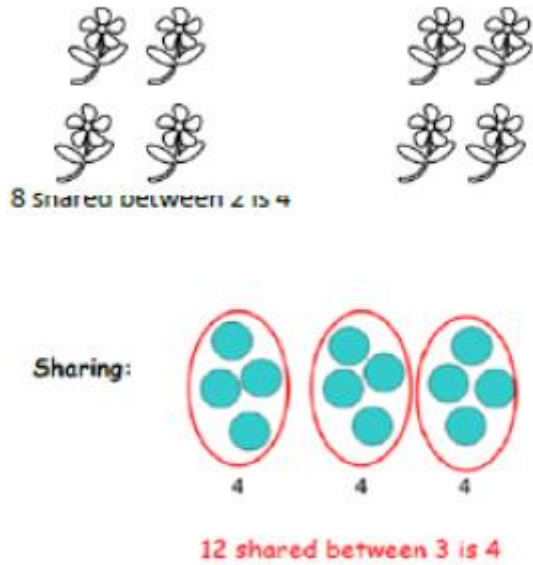
Scaling – to reduce a number, quantity or measurement by an amount

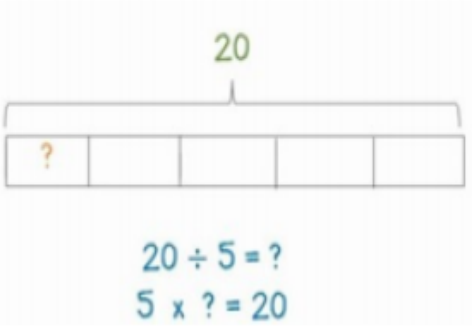
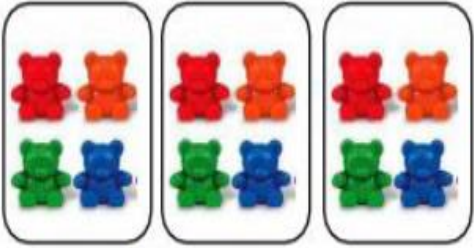
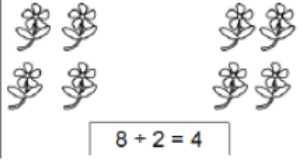
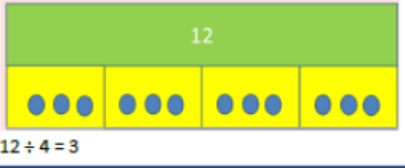
Short division – a method used to divide 2 or more digits by a 1 digit number

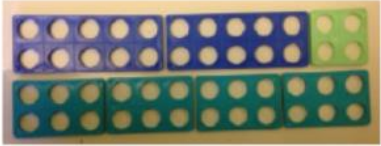
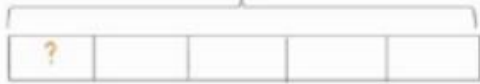
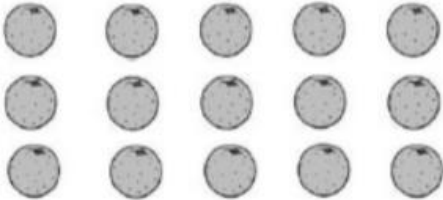
Y6 – Long division

Orders of operations – brackets, indices 2 3 $\sqrt{}$, multiplication and division, addition and subtraction

Objectives and Strategy:	Concrete	Pictorial	Abstract	Notes
EYFS				
To distribute objects between two people equally.				
To group a collection of objects into equal groups.	 Physically move the objects into groups.	 Draw lines around the objects, dividing them into equal groups.		Worth having a discussion here about identifying odd and even numbers: if the group divides equally, we call this number 'even'. If there are any left over from a set that's been shared equally, we call that number 'odd'.

Objectives and Strategy:	Concrete	Pictorial	Abstract	Notes
Year One				
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4 Sharing: 12 shared between 3 is 4	12 shared between 3 is 4.	

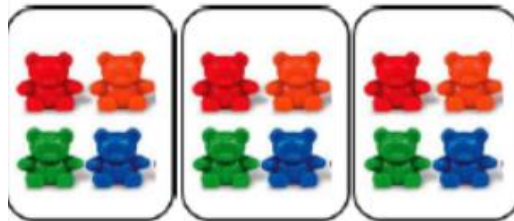
Objectives and Strategy:	Concrete	Pictorial	Abstract	Notes
Year Two				
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. 	Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	$20 \div 5 = 4$	
Division by sharing	 Use a variety of concrete objects to show equal sharing. This is a physical part-whole model.	Children use pictures or shapes to share quantities.  Children use bar modelling to show and support understanding. 	$12 \div 4 = 3$	

Objectives and Strategy	Concrete	Pictorial	Abstract	Notes
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 $96 \div 3 = 32$ 	Continue to use bar modelling to aid solving division problems.  $20 \div 5 = ?$ $5 \times ? = 20$	How many groups of 6 in 24? $24 \div 6 = 4$	
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$	

Division
with
remainders

$$14 \div 3 =$$

Divide objects between groups and
see how much is left over



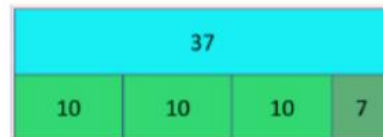
Jump forward in equal jumps on a number line
then see how many more you need to jump to
find a remainder.



Draw dots and group them to divide an amount
and clearly show a remainder.



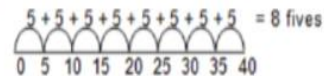
Use bar models to show division with remainders.



Example without remainder:

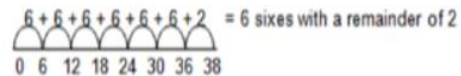
$$40 \div 5$$

Ask "How many 5s in 40?"



Example with remainder:

$$38 \div 6$$

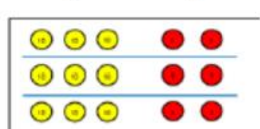
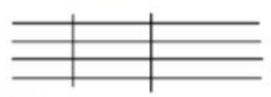
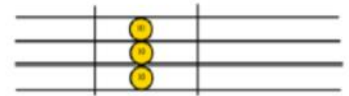
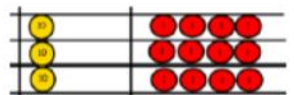
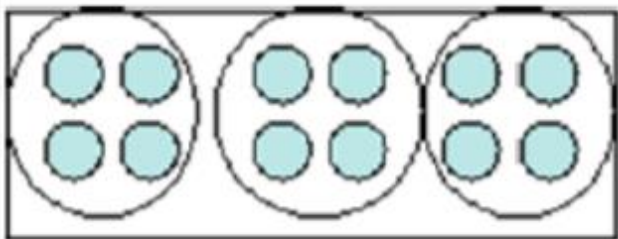
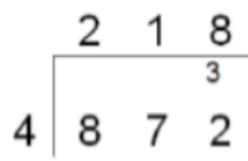
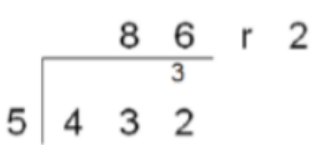
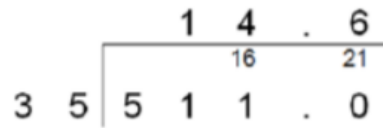
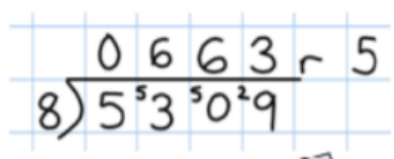


For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

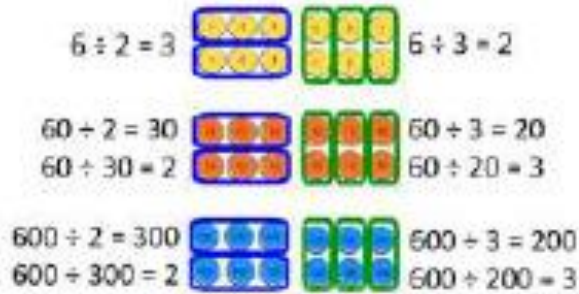
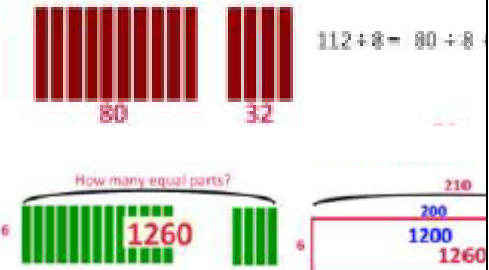
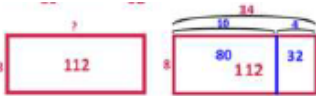
Complete written divisions and show the re-
mainder using r.

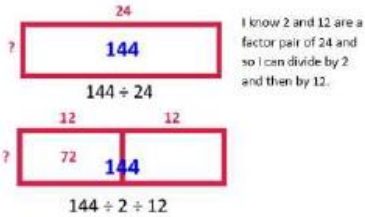
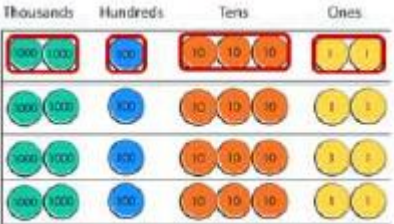
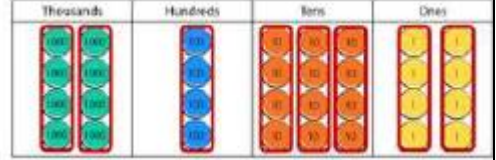
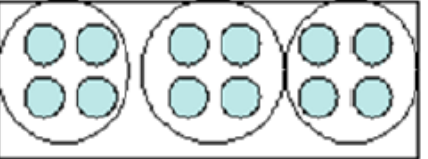
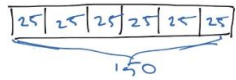
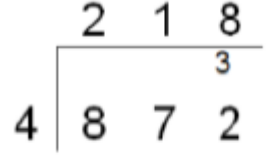
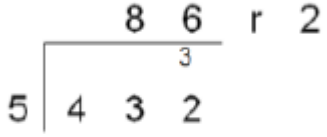
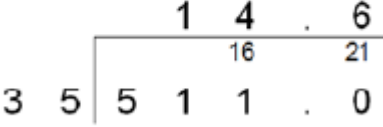
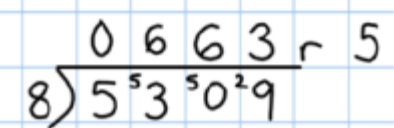
$$29 \div 8 = 3 \text{ REMAINDER } 5$$

$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$
 dividend divisor quotient remainder

Objectives and Strategy	Concrete	Pictorial	Abstract	Notes
Year Four				
	$96 \div 3$ Tens Units 3 2  Use place value counters to divide using the bus stop method alongside  $42 \div 3 =$ Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.   We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	https://youtu.be/fq2UXvSMWfs Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.  	

Year 5/6 Division

Objectives	concrete	Pictorial	Abstract	Notes
Year Five				
Year Six				
Deriving facts from known facts				Pupils use their growing knowledge of multiplication facts place value and derived facts to multiply mentally understanding the inverse relationship between multiplication and division allows corresponding division facts to be derived.
Using knowledge of multiples			$112 \div 8 = 80 \div 8 + 32 \div 8$	

Using knowledge of factors to divide				Pupils explore this strategy when dividing. $2 \times 2 = 4$ and so if you divide by 4 you can be achieved by dividing by two
Objectives 5/6	Concrete	Pictorial	Abstract	Notes
Short division dividing 4-digit numbers by 1-digit numbers	Sharing  Grouping 	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Using the part whole model below, how can you divide 615 by 5 without using short division?  There are 150 students. I want to split them up into 6 groups. How many students would be in each group? $150 \div 6 = 25$ 	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.  	The thought process of the teacher follows: How many 4s in 8? 2 How many 4s in 5? 1 with 1 remaining How many 4s in 12? 3 How many 4s in 8? 2 Warning: If you simply apply each step, the thinking goes wrong at each group. How many 4s in 8000? 2000 How many 4s in 500? 100 with 1 remaining (illogical) 125. Sharing the dividend builds confidence but however doesn't scaffold the thinking process. Using place value counters and the divisor for each power of ten to build understanding of the short division method. Area models are also useful when used with other strategies and examples.

$$46 \div 3$$

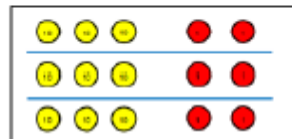
Tens

Units

3

2

3



Use place value counters to divide using the bus stop method alongside



Calculations
 $42 \div 3$

$$42 \div 3 =$$

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over

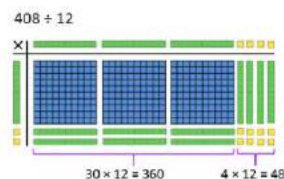


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Divide a 4-digit number by a 2-digit number



1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \end{array}$ Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ - 4 \\ \hline 1 \end{array}$ To find it, multiply $2 \times 2 = 4$, write that 4 under the five, and subtract to find the remainder of 1 ten.	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ - 4 \downarrow \\ \hline 18 \end{array}$ Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ - 4 \\ \hline 18 \end{array}$ Divide 2 into 18. Place 9 into the quotient.	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ - 4 \\ \hline 18 \\ - 18 \\ \hline 0 \end{array}$ Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract.	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ - 4 \\ \hline 18 \\ - 18 \\ \hline 0 \end{array}$ The division is over since there are no more digits in the dividend. The quotient is 29.

Notes – If dividing by a number outside of their known number facts pupils should start by recording some multiples of that number.

32

1

64

2

96

3